

Comment on Reparametrization Invariance of Quark-Lepton Complementarity

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We study the complementarity between quark and lepton mixing angles (QLC), the sum of an angle in quark mixing and the corresponding angle in lepton mixing is $\pi/4$. Experimentally in the standard PDG parametrization, two such relations exist approximately. These QLC relations are accidental which only manifest themselves in the PDG parametrization. We propose reparametrization invariant expressions for the complementarity relations in terms of the magnitude of the elements in the quark and lepton mixing matrices. In the exact QLC limit, it is found that $|V_{us}/V_{ud}| + |V_{e2}/V_{e1}| + |V_{us}/V_{ud}||V_{e2}/V_{e1}| = 1$ and $|V_{cb}/V_{tb}| + |V_{\mu3}/V_{\tau3}| + |V_{cb}/V_{tb}||V_{\mu3}/V_{\tau3}| = 1$. Expressions with deviations from exact complementarity are obtained. Implications of these relations are also discussed.

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Mixing between different generations of fermions in weak interaction is one of the most interesting issues in particle physics. The mixing is described by an unitary matrix in the charged current interaction of W-boson in the mass eigen-state of fermions. Quark mixing is described by the Cabibbo [1]-Kobayashi-Maskawa [2](CKM) matrix V_{CKM} , and lepton mixing is described by the Pontecorvo [3]-Maki-Nakawaga-Sakata [4] (PMNS) matrix V_{PMNS} with

$$L = -\frac{g}{\sqrt{2}}\bar{U}_L\gamma^\mu V_{\text{CKM}}D_LW_\mu^+ - \frac{g}{\sqrt{2}}\bar{E}_L\gamma^\mu V_{\text{PMNS}}N_LW_\mu^- + H.C. , \quad (1)$$

where $U_L = (u_L, c_L, t_L, \dots)^T$, $D_L = (d_L, s_L, b_L, \dots)^T$, $E_L = (e_L, \mu_L, \tau_L, \dots)^T$, and $N_L = (\nu_1, \nu_2, \nu_3, \dots)^T$ are the left-handed fermion generations. For n-generations, $V = V_{\text{CKM}}$ or V_{PMNS} is an $n \times n$ unitary matrix.

A commonly used form of mixing matrix for three generations of fermions is given by [5, 6],

$$V_{PDG} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ are the mixing angles and δ is the CP violating phase. This is the so called standard PDG parametrization. If neutrinos are of Majorana type, for the PMNS matrix one should include an additional diagonal matrix with two Majorana phases $\text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ multiplied to the matrix from right in the above. The two CP violating Majorana phases do not affect neutrino oscillations. To distinguish different CP violating phases, the phase δ is sometimes called Dirac CP violating phase. In our later discussions, we will indicate the mixing angles with superscriptions Q and L for quark and lepton sectors respectively when specification is needed.

There are a lot of experimental data on the mixing patterns in both the quark and lepton sectors. For quark mixing, the ranges of the magnitudes of the CKM matrix elements have been very well determined with [6]

$$\begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.00011}_{-0.00007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.00011}_{-0.00007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}. \quad (3)$$

From the above, we obtain the ranges for mixing angles θ_{ij}^Q ,

$$\theta_{12}^Q = 13.021^\circ \pm 0.039^\circ, \quad \theta_{23}^Q = 2.350^\circ \pm 0.052^\circ, \quad \theta_{13}^Q = 0.199^\circ \pm 0.008^\circ. \quad (4)$$

The CP violating phase has also been determined with $\delta^Q = 68.9^\circ$ [6].

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Considerable experimental data on lepton mixing have also been accumulated including the recent data from DAYA-BAY collaboration[8]. The global, $1\sigma(3\sigma)$, fit from neutrino oscillation data pre-DAYA-Bay data gives [7],

$$\begin{pmatrix} 0.824^{+0.011}_{-0.010}({}^{+0.032}_{-0.032}) & 0.547^{+0.016}_{-0.014}({}^{+0.047}_{-0.044}) & 0.145^{+0.022}_{-0.031}({}^{+0.065}_{-0.113}) \\ 0.500^{+0.027}_{-0.021}({}^{+0.076}_{-0.071}) & 0.582^{+0.050}_{-0.023}({}^{+0.139}_{-0.069}) & 0.641^{+0.061}_{-0.023}({}^{+0.168}_{-0.063}) \\ 0.267^{+0.044}_{-0.027}({}^{+0.123}_{-0.088}) & 0.601^{+0.048}_{-0.022}({}^{+0.133}_{-0.069}) & 0.754^{+0.052}_{-0.020}({}^{+0.143}_{-0.054}) \end{pmatrix}. \quad (5)$$

and the mixing angles are given by [7]

$$\begin{aligned} \theta_{12}^L &= 33.59^\circ \pm 1.02^\circ (\pm 3.05^\circ), & \theta_{23}^L &= 40.40^\circ \pm 3.17^\circ (\pm 8.63^\circ), \\ \theta_{13}^L &= 8.33^\circ \pm 1.39^\circ (\pm 3.57^\circ). \end{aligned} \quad (6)$$

The recent measured non-zero θ_{13} at a 5.2σ level by the DAYA-BAY collaboration is[8] $\sin^2(2\theta_{13}) = 0.092 \pm 0.016(stat.) \pm 0.005(syst.)$. Translating into the angle θ_{13} , it is $\theta_{13} = 8.8^\circ \pm 0.8^\circ$. This values agrees with global fit value very well, but with a smaller error bar. At present there is no experimental data on the CP violating Dirac phase δ^L and Majorana phases α_i .

The mixing angles for quark and lepton sectors, a priori, are unrelated. If there is a way to connect the seemingly independent mixing angles in these two sectors, it would gain more insights about fermion mixing. Indeed there is a very nice way to make the connection via the so called quark-lepton complementarity (QLC) [9–12]. Here the QLC relations refer to

$$\theta_{12}^Q + \theta_{12}^L = \frac{\pi}{4}, \quad \theta_{23}^Q + \theta_{23}^L = \frac{\pi}{4}. \quad (7)$$

The third angles are approximately zero, $\theta_{13}^Q \sim \theta_{13}^L \sim 0$.

The first two relations hold within experimental errors,

$$\theta_{12}^Q + \theta_{12}^L = 46.606^\circ \pm 1.019^\circ, \quad \theta_{23}^Q + \theta_{23}^L = 42.746^\circ \pm 3.171^\circ. \quad (8)$$

At present the above relations are still at the phenomenological level. It is far from having a complete theoretical understanding although there are attempt to build theoretical models [12]. Even at the phenomenological level, there are some questions to address about these relations. One of them is that there are different ways to parameterize the mixing matrices. The relation between the angles may only hold in a particular parametrization [13]. Therefore these relations seem to be parametrization dependent. Let us illustrate this point by working out the values of the angles in the original KM parametrization [2],

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}. \quad (9)$$

Using the observed values for the mixing matrices, one would obtain

$$\begin{aligned} \theta_1^Q &= 13.023^\circ \pm 0.038^\circ, & \theta_2^Q &= 2.192^\circ \pm 0.059^\circ, & \theta_3^Q &= 0.882^\circ \pm 0.036^\circ \\ \theta_1^L &= 34.485^\circ \pm 1.028^\circ, & \theta_2^L &= 28.086^\circ \pm 3.762^\circ, & \theta_3^L &= 14.830^\circ \pm 2.423^\circ. \end{aligned} \quad (10)$$

We see that only $\theta_1^Q + \theta_1^L$ is close to $\pi/4$, and the other two angles sums do not have the complementarity relations.

It has been shown that there are nine independent ways to parameterize the mixing matrices [14]. We have checked, in details, whether similar complementarity relations hold in some of the other parameterizations. We find that only the PDG parametrization has the complementarity property for two different angles. To avoid the complementarity relations be parametrization dependent, it would be more meaningful to find the relations using quantities which are reparametrization invariant. We will work the scenario that there are two complementarity relations discussed above. To this end we find the magnitudes of the elements in the mixing matrices convenient quantities. They can be used to represent the complementarity relations. In the following we derive such relations.

We will take the usual complementarity relations in eq.(7) as the starting point. By taking sine and cosine on both sides, we obtain

$$\begin{aligned} \sin(\theta_{12}^Q + \theta_{12}^L) &= \sin \theta_{12}^Q \cos \theta_{12}^L + \cos \theta_{12}^Q \sin \theta_{12}^L = \frac{1}{\sqrt{2}}, \\ \cos(\theta_{12}^Q + \theta_{12}^L) &= \cos \theta_{12}^Q \cos \theta_{12}^L - \sin \theta_{12}^Q \sin \theta_{12}^L = \frac{1}{\sqrt{2}}; \\ \sin(\theta_{23}^Q + \theta_{23}^L) &= \sin \theta_{23}^Q \cos \theta_{23}^L + \cos \theta_{23}^Q \sin \theta_{23}^L = \frac{1}{\sqrt{2}}, \\ \cos(\theta_{23}^Q + \theta_{23}^L) &= \cos \theta_{23}^Q \cos \theta_{23}^L - \sin \theta_{23}^Q \sin \theta_{23}^L = \frac{1}{\sqrt{2}}. \end{aligned} \quad (11)$$

Using the relations between the angles and elements in the mixing matrices, and taking the ratios of the first two and the last two equations above, we have

$$\begin{aligned}\frac{\tan \theta_{12}^Q + \tan \theta_{12}^L}{1 - \tan \theta_{12}^Q \tan \theta_{12}^L} &= \frac{|V_{us}/V_{ud}| + |V_{e2}/V_{e1}|}{1 - |V_{us}/V_{ud}||V_{e2}/V_{e1}|} = 1, \\ \frac{\tan \theta_{23}^Q + \tan \theta_{23}^L}{1 - \tan \theta_{23}^Q \tan \theta_{23}^L} &= \frac{|V_{cb}/V_{tb}| + |V_{\mu 3}/V_{\tau 3}|}{1 - |V_{cb}/V_{tb}||V_{\mu 3}/V_{\tau 3}|} = 1.\end{aligned}\quad (12)$$

The complementarity relations can now be written as

$$\begin{aligned}\frac{|V_{us}|}{|V_{ud}|} + \frac{|V_{e2}|}{|V_{e1}|} + \frac{|V_{us}|}{|V_{ud}|} \frac{|V_{e2}|}{|V_{e1}|} &= 1, \\ \frac{|V_{cb}|}{|V_{tb}|} + \frac{|V_{\mu 3}|}{|V_{\tau 3}|} + \frac{|V_{cb}|}{|V_{tb}|} \frac{|V_{\mu 3}|}{|V_{\tau 3}|} &= 1.\end{aligned}\quad (13)$$

The above relations are the new reparametrization invariant complementarity relations. These relations may tell more about the mixing matrix elements. One can solve these relations to express ratios of elements of lepton mixing matrix in terms of the ratios of elements of quark mixing matrix to obtain

$$\frac{|V_{e2}|}{|V_{e1}|} = \frac{1 - |V_{us}|/|V_{ud}|}{1 + |V_{us}|/|V_{ud}|}, \quad \frac{|V_{\mu 3}|}{|V_{\tau 3}|} = \frac{1 - |V_{cb}|/|V_{tb}|}{1 + |V_{cb}|/|V_{tb}|}.\quad (14)$$

Similarly one can express ratios of quark mixing matrix elements in terms of the lepton mixing matrix elements.

Experimentally, the quark mixing matrix elements are more precisely known, therefore one can take them as input to predict the lepton mixing matrix element. We have

$$\frac{|V_{e2}|}{|V_{e1}|} = 0.624369 \pm 0.00095, \quad \frac{|V_{\mu 3}|}{|V_{\tau 3}|} = 0.921165 \pm 0.00166.\quad (15)$$

Notice that $|V_{\tau 3}| > |V_{\mu 3}|$. This is consistent with data from neutrino oscillation at 2σ and 1σ level for the first and the second ratios in the above equations, respectively.

The tribimaximal mixing is a good approximation for the lepton mixing matrix [15]. Data from global fit and the recent DAYA-BAY measurement show that the tribimaximal mixing pattern, which predicts $\theta_{13} = 0$, has to be modified. There are models prefer that the elements in the second column to be all equal to $1/\sqrt{3}$ [16]. If this is true then combining $V_{e2} = 1/\sqrt{3}$ and eq.(15), one would obtain $|V_{e1}|^2 + |V_{e2}|^2 > 1$. This indicates that the QLC is not consistent with this type of models. There are also speculations that other columns or row in the tribimaximal mixing is kept unaltered but other elements are modified [16]. If one keeps the first column of the tribimaximal mixing matrix unaltered, one would have $V_{e1} = 2/\sqrt{6}$. Coming with eq.(15), one would obtain $V_{e3} = 0.27$. This predicts too large a V_{e3} outside the 1σ region allowed by the pre-DAYA-BAY global data fit and more from present DAYA-BAY data. Also if one keeps the second row of tribimaximal mixing unaltered, one would then predict $|V_{\mu 3}|^2 + |V_{\tau 3}|^2 > 1$ which is not allowed. If one keeps the third row of tribimaximal mixing unaltered [17], then one would obtain also too large a $V_{e3} = 0.275$. Satisfaction of QLC requirement would require modification of the tribimaximal mixing significantly [18].

In the above relations, the elements V_{e3} and V_{ub} do not show up directly. To have some information about the 13 entries of the mixing matrices, let us consider the unitarity of the first row and the third column. Eq.(14) suggest that one can write

$$\begin{aligned}|V_{e1}| &= \frac{a}{\sqrt{2}}(|V_{ud}| + |V_{us}|), \quad |V_{e2}| = \frac{a}{\sqrt{2}}(|V_{ud}| - |V_{us}|), \\ |V_{\tau 3}| &= \frac{b}{\sqrt{2}}(|V_{tb}| + |V_{cb}|), \quad |V_{\mu 3}| = \frac{b}{\sqrt{2}}(|V_{tb}| - |V_{cb}|).\end{aligned}\quad (16)$$

One then obtains

$$\sum_i |V_{ei}|^2 = a^2(|V_{ud}|^2 + |V_{us}|^2) + |V_{e3}|^2 = b^2(|V_{cb}|^2 + |V_{tb}|^2) + |V_{e3}|^2 = 1.\quad (17)$$

a and b are related by $b^2 = a^2(|V_{ud}|^2 + |V_{us}|^2)/(|V_{cb}|^2 + |V_{tb}|^2) = (1.0000 \pm 0.0004)a^2$.

The size for V_{e3} depends on what a in the form: $|V_{e3}| = \sqrt{1 - a^2(|V_{ud}|^2 + |V_{us}|^2)}$. Using current 1σ allowed value for $|V_{e3}|$, a is determined to be smaller than 0.99.

There may be modifications to the complementarity relations. The modifications can be written as $\theta^Q + \theta^L = \pi/4 + \alpha$. In this case the reparametrization invariant relations will be modified to

$$\begin{aligned} \frac{|V_{us}|}{|V_{ud}|} + \frac{|V_{e2}|}{|V_{e1}|} + \frac{|V_{us}|}{|V_{ud}|} \frac{|V_{e2}|}{|V_{e1}|} &= 1 + \left(1 - \frac{|V_{us}|}{|V_{ud}|} \frac{|V_{e2}|}{|V_{e1}|}\right) \frac{\tan \alpha_{12}}{1 - \tan \alpha_{12}}, \\ \frac{|V_{cb}|}{|V_{tb}|} + \frac{|V_{\mu 3}|}{|V_{\tau 3}|} + \frac{|V_{cb}|}{|V_{tb}|} \frac{|V_{\mu 3}|}{|V_{\tau 3}|} &= 1 + \left(1 - \frac{|V_{cb}|}{|V_{tb}|} \frac{|V_{\mu 3}|}{|V_{\tau 3}|}\right) \frac{\tan \alpha_{23}}{1 - \tan \alpha_{23}}. \end{aligned} \quad (18)$$

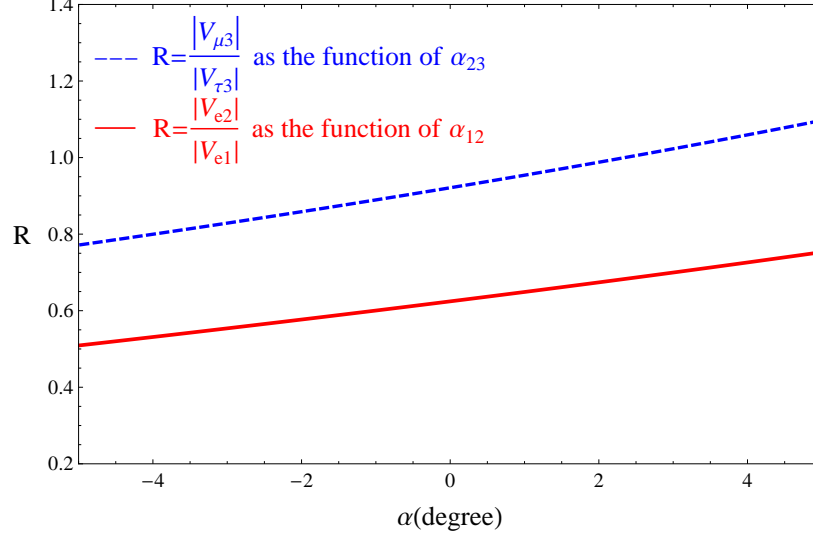


FIG. 1: Ratios of $|V_{e2}|/|V_{e1}|$ and $|V_{\mu 3}|/|V_{\tau 3}|$ as functions of deviations α_{12} and α_{23} , respectively.

Expressing lepton mixing matrix elements in terms of the quark mixing elements and the deviations, we have

$$\begin{aligned} \frac{|V_{e2}|}{|V_{e1}|} &= \frac{1 + \tan \alpha_{12} - |V_{us}|/|V_{ud}|(1 - \tan \alpha_{12})}{1 - \tan \alpha_{12} + |V_{us}|/|V_{ud}|(1 + \tan \alpha_{12})}, \\ \frac{|V_{\mu 3}|}{|V_{\tau 3}|} &= \frac{1 + \tan \alpha_{23} - |V_{cb}|/|V_{tb}|(1 - \tan \alpha_{23})}{1 - \tan \alpha_{23} + |V_{cb}|/|V_{tb}|(1 + \tan \alpha_{23})}. \end{aligned} \quad (19)$$

In Fig. 1 we show how $|V_{e2}|/|V_{e1}|$ and $|V_{\mu 3}|/|V_{\tau 3}|$ depend on α_{12} and α_{23} . We see that a small deviation away from the complementarity relation can cause sizeable difference in the predicted neutrino mixing matrix elements. With more precisely measured mixing matrix elements in both the quark and lepton sectors, the QLC and deviations can be studied more which may give some hints on theoretical models for quark and lepton mixing matrices.

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- [1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).
 - [2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
 - [3] B. Pontecorvo, Sov. Phys. JETP **6**, 429 (1957).
 - [4] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
 - [5] L.L. Chau and W.Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984).
 - [6] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. GG **37**, 075021 (2010).
 - [7] G. Fogli *et al.*, Phys. Review. **D84**, 053007(2011).
 - [8] F.P.An *et al.* [DAYA-BAY Collaboration], arXiv:1203.1669[hep-ex].
 - [9] A.Y. Smirnov, Talk given at 2nd International Workshop on Neutrino Oscillations in Venice (NO-VE 2003), Venice, Italy, 3-5 Dec 2003. Published in *Venice 2003, Neutrino oscillations* 1-2, arXiv:hep-ph/0402264.
 - [10] H. Minakata and A.Y. Smirnov, Phys. Rev. **D70**, 073009 (2004); M. Raidal, Phys. Rev. Lett. **93**, 161801 (2004).
 - [11] N. Li and B. Q. Ma, Phys. Rev. D **71**, 097301 (2005) [arXiv:hep-ph/0501226]; N. Li and B. Q. Ma, Eur. Phys. J. C **42**, 17 (2005) [arXiv:hep-ph/0504161]; B. C. Chauhan, M. Picariello, J. Pulido and E. Torrente-Lujan, Eur. Phys. J. C **50**, 573 (2007) [arXiv:hep-ph/0605032]; S. W. Li and B. Q. Ma, Phys. Rev. D **77**, 093005 (2008) [arXiv:0806.0670 [hep-ph]];

- P. H. Frampton and S. Matsuzaki, *Mod. Phys. Lett. A* **24**, 429 (2009) [arXiv:0807.4785 [hep-ph]]; Z. z. Xing, *Phys. Lett. B* **679**, 111 (2009) [arXiv:0904.3172 [hep-ph]]; Y. H. Ahn, H. Y. Cheng and S. Oh, *Phys. Rev. D* **83**, 076012 (2011) [arXiv:1102.0879 [hep-ph]]; S. K. Kang, *Phys. Rev. D* **83**, 097301 (2011) [arXiv:1104.1969 [hep-ph]]; N. Qin and B. Q. Ma, *Phys. Lett. B* **702**, 143 (2011) [arXiv:1106.3284 [hep-ph]].
- [12] P.H. Frampton and R.N. Mohapatra, *JHEP* **0501**, 025 (2005); J. Ferrandis and S. Pakvasa, *Phys. Rev. D* **71**, 033004 (2005); S.K. Kang, C.S. Kim and J. Lee, *Phys. Lett. B* **619**, 129 (2005); S. Antusch, S.F. King and R.N. Mohapatra, *Phys. Lett. B* **618**, 150 (2005); Z. z. Xing, *Phys. Lett. B* **618**, 141 (2005)[arXiv:hep-ph/0503200]; M.A. Schmidt and A.Y. Smirnov, *Phys. Rev. D* **74**, 113003 (2006); F. Plentinger, G. Seidl and W. Winter, *Phys. Rev. D* **76**, 113003 (2007); W. Winter, *AIP Conf. Proc.* **981**, 199 (2008) [arXiv:0710.5660 [hep-ph]]; K. M. Patel, *Phys. Lett. B* **695**, 225 (2011) [arXiv:1008.5061 [hep-ph]].
- [13] C. Jarlskog, *Phys. Lett. B* **625**, 63 (2005) [arXiv:hep-ph/0507212]; Y. j. Zheng, *Phys. Rev. D* **81**, 073009 (2010) [arXiv:1002.0919 [hep-ph]].
- [14] H. Fritzsch and Z. -z. Xing, *Phys. Rev. D* **57**, 594(1998).
- [15] P.F. Harrison, D.H. Perkins, and W.G. Scott, *Phys. Lett. B* **458**, 79 (1999); *Phys. Lett. B* **530**, 167 (2002); Z.Z. Xing, *Phys. Lett. B* **533**, 85 (2002); P.F. Harrison and W.G. Scott, *Phys. Lett. B* **535**, 163 (2002); *Phys. Lett. B* **557**, 76 (2003); X.-G. He and A. Zee, *Phys. Lett. B* **560**, 87 (2003). See also L. Wolfenstein, *Phys. Rev. D* **18**, 958 (1978); Y. Yamanaka, H. Sugawara, and S. Pakvasa, *Phys. Rev. D* **25**, 1895(1982); **29**, 2135(E)(1984).
- [16] X. G. He, Y. Y. Keum and R. R. Volkas, *JHEP* **0604**, 039 (2006) [arXiv:hep-ph/0601001]; Xiao-Gang He and A. Zee, *Phys. Lett. B* **645**, 427(2007), hep-ph/0607163; C. H. Albright and W. Rodejohann, *Eur. Phys. J. C* **62**, 599 (2009) [arXiv:0812.0436 [hep-ph]]; C. H. Albright, A. Dueck, W. Rodejohann, *Eur. Phys. J. C* **70**, 1099-1110 (2010). [arXiv:1004.2798 [hep-ph]]; Xiao-Gang He and A. Zee, *Phys. Rev. D* **84**, 053004(2011)[arXiv:1106.4359 [hep-ph]].
- [17] R. Friedberg and T.D. Lee, *Chin. Phys. C* **34**, 1547; Erratum-ibid. **34**, 1905(2011)[arXiv:1008.0453 [hep-ph]].
- [18] P. H. Frampton and S. Matsuzaki, *Mod. Phys. Lett. A* **24**, 429 (2009) [arXiv:0807.4785 [hep-ph]]; X. G. He, S. W. Li and B. Q. Ma, *Phys. Rev. D* **78**, 111301 (2008) [arXiv:0809.1223 [hep-ph]]; X. G. He, S. W. Li and B. Q. Ma, *Phys. Rev. D* **79**, 073001 (2009) [arXiv:0903.2880 [hep-ph]]; Y. Shimizu and R. Takahashi, *Europhys. Lett.* **93**, 61001 (2011) [arXiv:1009.5504 [hep-ph]].